Implications of $\mathcal{CP}$-violating transitions in hot quark matter on heavy ion collisions

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Abstract. Quantum Chromodynamics (QCD) predicts that topological charge changing transitions will take place in hot quark matter. Such transitions induce $\mathcal{P}$- and $\mathcal{CP}$-violating effects. We will show that in the presence of a magnetic field these transitions can separate quarks according to their electric charge along the direction of the magnetic field. This is the so-called Chiral Magnetic Effect. We will argue that it might be possible to observe the Chiral Magnetic Effect in heavy ion collisions.

1. Introduction

Although the ground state of QCD at zero temperature cannot break the parity ($\mathcal{P}$) and charge-parity ($\mathcal{CP}$) symmetries spontaneously [1], this is not necessarily true anymore at finite temperatures and/or chemical potentials [2]. Hence the possibility of spontaneous breakdown of discrete symmetries of space and time arises [3], which could be realized in heavy ion collisions [4]. In Ref. [5] it was argued that during the chiral symmetry breaking phase transition the matter produced in a heavy ion collision may cool to a meta-stable vacuum. This meta-stable vacuum can effectively be described by QCD with a finite θ angle. The decay of such a meta-stable vacuum could give rise to all kinds of interesting $\mathcal{P}$- and $\mathcal{CP}$-violating behavior [6], which might be detected in experiment [7] by using suitable observables [8].

Kharzeev has pointed out that if $\mathcal{P}$- and $\mathcal{CP}$-violating processes are taking place in the quark matter produced in heavy ion collisions, this will lead to separation of electric charge along the direction of angular momentum of the collision [9]. This in some sense similar to an electric dipole moment, but now the direction of the dipole moment is expected to fluctuate from event-to-event. Voloshin has shown that this effect can be studied experimentally by analyzing correlations between charged particles and the reaction plane [10]. Preliminary data of the STAR collaboration is presented in Ref. [11]. The scenario of Kharzeev [9] was worked out in more detail in Refs. [12, 13] and called the Chiral Magnetic Effect: topological charge changing transitions induce chirality which leads to separation of charge along the direction of the magnetic field. We will study this effect in detail in this article.

2. Generating chirality

We will assume that the quarks are massless. This is expected to be a reasonable approximation in the quark gluon plasma phase where the typical momenta of the quarks are much larger than their masses. In that case right-handed quarks and anti-quarks have spin and momentum in the same direction, while left-handed ones have them in the opposite direction. The projection of spin on the momentum is called helicity. Since we have assumed that the quark are massless this is the same as chirality.

Massless quarks can change their chirality by interacting with gluons. In QCD there is an exact relation, the so-called axial Ward-Identity which relates the chirality change to the properties of the gluon fields. This identity arises from the axial anomaly [14] and reads

$$(N_L - N_R)(t = \infty) - (N_L - N_R)(t = -\infty) = 2Q_{\mathrm{w}}. \qquad (1)$$

In this equation $N_{L,R}$ stands for the total number of left/right-handed quarks plus anti-quarks of a particular flavor in the background of a certain gluon field. The change of chirality of the quarks is equal to twice the winding number Q_{w} of the gluon fields. This winding number can be computed as follows

$$Q_{\mathrm{w}} = \frac{g^2}{32\pi^2} \int \mathrm{d}^4x \, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a. \qquad (2)$$

Here g denotes the QCD coupling constant with generators normalized as $\mathrm{tr}\, t_a t_b = \delta_{ab}/2$. The gluonic field tensor and its dual are respectively $F^a_{\mu\nu}$ and $\tilde{F}^a_{\mu\nu} = \epsilon_{\mu\nu}{}^{\rho\sigma} F^a_{\rho\sigma}/2$. For gluon fields which go to a vacuum solution at $t = \pm\infty$ the winding number Q_{w} is an integer as we will argue in the next section.

In perturbative QCD it is impossible to change the chirality of massless quarks. This can be easily inferred from Eq. (1) since in perturbative QCD one only takes into account gluon fields with $Q_{\mathrm{w}} = 0$. This is a good approximation at very high energies where the strong coupling constant α_S is small, but as we will see in more detail in the next section, in the non-perturbative regime the gluon fields which a nonzero winding number can give a significant contribution to physical quantities. One famous example is the mass of the η' meson. A very important conclusion we can draw now from Eq. (1) is that chirality change and hence $\mathcal{P}$- and $\mathcal{CP}$-violation are directly linked to the topology of the gluon fields. If one observes a difference between the number of left- and right-handed fermions, this immediately tells us that $\mathcal{P}$ and $\mathcal{CP}$ are violated on an event-by-event basis. Moreover it will be a direct proof the existence of topologically non-trivial gluon fields.

3. Generating winding number

Now how can gluon fields wind, and why? In order to answer that question let us for a moment forget about the quarks and have look to the vacuum structure of a pure SU(3) gauge theory. In the vacuum the energy density of the gauge fields is minimal,

which implies that the gauge fields are static and have to be a pure gauge. In the temporal gauge ($A_0 = 0$) one then finds that $A_i(\boldsymbol{x}) = ig^{-1}U(\boldsymbol{x})\partial_i U^\dagger(\boldsymbol{x})$, where $U(\boldsymbol{x})$ is an element of the gauge group SU(3). It is now possible to assign to each classical vacuum a topological invariant, the Chern-Simons number $N_{\rm CS}$ [15]. For the vacua this number is an integer and given by

$$N_{\rm CS} = \frac{1}{24\pi^2}\int \mathrm{d}^3x\, \epsilon^{ijk}\mathrm{tr}\left[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\right]. \tag{3}$$

We have illustrated this in Fig. 1. The different vacua are separated by an energy barrier of order $\Lambda_{\rm QCD}$. A gauge field configuration with a certain winding number $Q_{\rm w}$ interpolates between two classical vacua. One can show that

$$Q_{\rm w} = N_{\rm CS}(t=\infty) - N_{\rm CS}(t=-\infty). \tag{4}$$

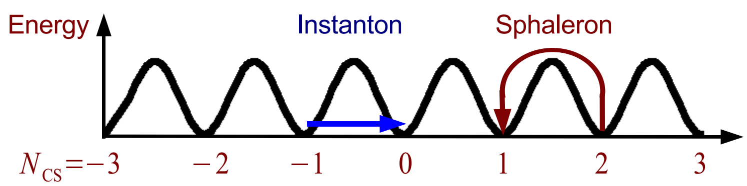

Figure 1. Illustration of the vacuum structure of an SU(3) Yang-Mills theory. The different vacua can be characterized by a Chern-Simons number $N_{\rm CS}$, which is an integer. An instanton interpolates from one vacuum to another by tunneling, whereas a sphaleron does this via hopping over this barrier.

At zero temperature the only possibility to go from one vacuum to another is by tunneling through the potential barrier. The gauge field configuration responsible for such a tunneling process is called an instanton [16]. The rate is exponentially suppressed, even at finite temperature [17].

At finite temperature another possibility arises, which is hopping over the barier. The gauge field configuration which just hops over the barier is called a sphaleron [18]. The rate is not exponentially suppressed. The rate Γ has been computed on the lattice for SU(2) Yang-Mills at high temperatures [19]. Extrapolating this result to SU(3) gives

$$\Gamma = \frac{\mathrm{d}N}{\mathrm{d}^3\mathrm{x}\,\mathrm{dt}} \approx 386\alpha_S^5 T^4. \tag{5}$$

If the density of quarks is small, the rate will not be changed much when massless quarks are taken into account. Hence QCD predicts that in a thermalized quark gluon plasma several sphaleron transitions per fm^3 per fm/c will take place.

In our estimations for the amount of $\mathcal{P}$ and $\mathcal{CP}$ violation in heavy ion collisions we will use the sphaleron rate. This is correct if a quark gluon plasma has been formed. But as was shown in [20] the initial state of the matter produced in a heavy ion collision, the so-called glasma, is also capable of generating differences in the Chern-Simons number. Kinetic theory descriptions of an evolving quark gluon plasma find variations in the Chern-Simons number [21] too. Ultimately one hopes to understand quantitatively how

the glasma evolves into the quark gluon plasma. This then could give us a more reliable estimate of the amount of chirality change for very early times.

4. The Chiral Magnetic Effect

In order to observe $\mathcal{P}$ and $\mathcal{CP}$ violation on an event-by-event basis in heavy ion collisions we have to know how to distinguish left- from right-handed quarks. Measuring directly the helicity of the quarks is impossible since one only measures hadrons in the detectors. The solution is polarization, which, as we will see in the next section, can be generated by the (electromagnetic) magnetic field created by the colliding ions.

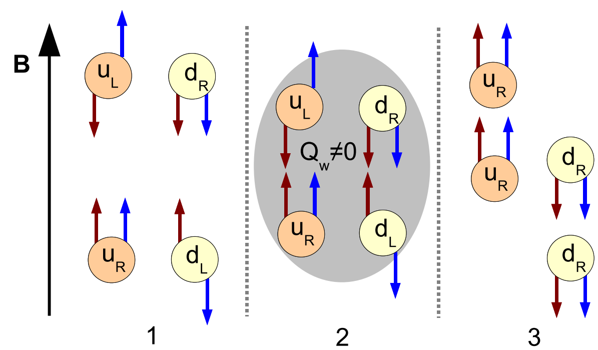

Figure 2. Illustration of the Chiral Magnetic Effect in a very large homogeneous magnetic field. The red arrows denote the direction of momentum, the blue arrows – the spin of the quarks. (1) The magnetic field polarizes the quarks. (2) The quarks interact with gluon fields which have $Q_\mathrm{w} = -1$ in this case. (3) The chirality change induces an electromagnetic current along the direction of the magnetic field.

Let us for a moment assume that we have a homogeneous magnetic field B pointing in the z-direction. Let us furthermore assume that the magnitude of B is much larger than the square of the typical momentum of the quarks. In that case the quarks will be fully polarized along the magnetic field. This means that the spins of the quarks align along the direction of the magnetic field depending on their electric charge. But since the quarks are massless, the momenta of the quarks will also align along the direction of the magnetic field. We have illustrated this situation in Fig. 2. The left- and right-handed quarks will now move in opposite directions, hence we can distinguish them.

After the quarks have interacted with the gluon fields a difference between the number of left- and right-handed quarks is generated which is equal to $2Q_\mathrm{w}$. Because left- and right-handed quarks move in opposite directions, an electromagnetic current is set up along the magnetic field. This is the Chiral Magnetic Effect.

If the electromagnetic current flows in a finite volume, a charge difference of magnitude $Q = 2Q_\mathrm{w}\sum_f |q_f|$ will be generated between the upper and lower hemisphere. Here q_f denotes the charge of a quark with flavor f. This charge difference is the same if antiquarks change their chirality or when quark-antiquark pairs are produced from a gluon field with a winding number.

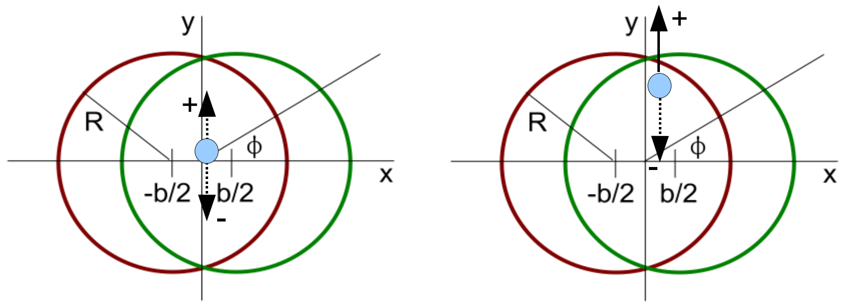

Figure 3. Two cross-sectional views of a non-central heavy ion collision along the beam-axis (z-axis). The plane $y = 0$ is called the reaction plane. The region in which the two nuclei overlap contains the participants, the regions in which they do not contain the spectators. Two examples of sphaleron transitions (indicated with a small circle) are depicted. The contribution to the separation of charge of the transitions near the center (left-hand side) is suppressed with respect to transitions near the surface (right-hand side) due to screening.

If the magnetic field is moderate, quarks with high momentum will be less polarized. Let us denote the degree of polarization of a quark as $\gamma(q_f)$. The expectation value of the induced charge difference will now become $Q = 2Q_{\mathrm{w}} \sum_f |q_f|\gamma(q_f)$. Only quarks which have momenta smaller than the inverse size $1/\rho$ of the gauge field configurations with winding number Q_{w} will interact, and hence change their helicity. Therefore only the polarization of quarks with momentum smaller than $1/\rho$ is relevant. In [13] we found that the polarization can be estimated by the following formula $\gamma(q_f) \approx 2|q_f eB|\rho^2$.

The typical size of a sphaleron is bounded by the chromomagnetic screening length of the quark gluon plasma which is $\rho \sim 1/(\alpha_S T)$. Hence in order to get reasonable polarization the magnetic field has to be of order $\alpha_s^2 T^2$, which is $10^3 - 10^4$ MeV^2. This corresponds to $10^{13} - 10^{14}$ T. In the next section we will see that such huge fields are created in heavy ion collisions.

5. The implications for heavy ion collisions

For reference we have displayed a cross-sectional view of a heavy ion collision in Fig. 3. If two heavy ions collide enormous magnetic fields are generated in the direction of angular momentum (the y-direction). We have computed the magnetic field generated by the spectators and participants using a classical calculation which included the effect of baryon stopping in the participant region [13]. The results are displayed in Fig. 4. The magnetic field is so large because of the large charge of the nuclei and the short distances and time scales involved. The magnetic field is capable of polarizing quarks to a certain degree just after the collision.

Because the quarks will be polarized along the y-axis, the Chiral Magnetic Effect will separate quarks along this axis. Now imagine a sphaleron transition taking place in the center of the participant region (left-hand side of Fig. 3). The quarks that are being

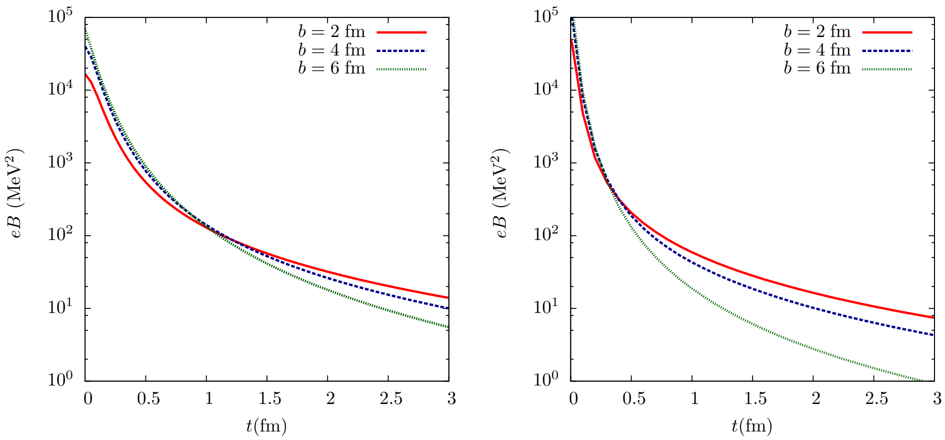

Figure 4. Magnetic field at the center of a Gold-Gold collision, for different impact parameters. In left plot the center of mass energy is 62 GeV per nucleon pair, in the right plot it is 200 GeV per nucleon pair.

separated would still experience interactions, so that presumably their momenta get randomized. If the sphaleron transition takes place at near the surface of the participant region (right-hand side of Fig. 3), a quark with one kind of charge is able to hadronize without experience many interactions, while the quark with opposite charge still has to travel through the hot medium. Since charge is conserved in hadronization, asymmetries in charge generated by quarks, will end up into asymmetries in charged hadrons.

Let us now define $\Delta_\pm$ to be the difference between the amount of positive / negative charge above and below the reaction plane. We have argued in Ref. [13] that every time a topological charge changing transition is made, $\Delta_\pm$ is modified as follows

$$\Delta_+ \to \Delta_+ \pm \sum_f |q_f|\gamma(q_f)\xi_\pm(x_\perp), \qquad \Delta_- \to \Delta_- \mp \sum_f |q_f|\gamma(q_f)\xi_\mp(x_\perp). \quad (6)$$

Here $\xi_\pm(x_\perp)$ is a phenomenological screening function. By folding rate of sphaleron transitions with the square of the changes from Eq. (6) and integrating over time and the volume of the participant region it is possible to compute the variance of $\Delta_\pm$ and the correlation $\langle\Delta_+\Delta_-\rangle$ [13]. Since the magnetic field decreases rapidly as a function of time, the main contribution to the correlations comes from early times.

In [13] we have studied the magnitude of $a_{ij} \sim \langle\Delta_i\Delta_j\rangle/(N_iN_j)$. Here $i, j = \pm$ and $N_\pm$ denotes the total number of particles of a particular charge. We have estimated that a_{ij} for large impact parameters can be of order 10^{-4}, with order of magnitude uncertainties. This means that the typical expected asymmetries could be of order 1%. We expect that a_{ij} will increase as a function of impact parameter, since for larger impact parameters the magnetic field is larger and the screening is relatively less important. Since a_{+-} is affected more by the screening effect, $|a_{+-}/a_{++}|$ is expected to grow as a function of impact parameter.

The correlators a_{ij} can be measured using the method proposed by Voloshin [10]. In order to obtain the correlators one has to average over many events. The observables

itself are not $\mathcal{P}$- and $\mathcal{CP}$-odd, in a sense one is measuring the absolute value of the dipole moment. Preliminary data from the STAR collaboration which is presented in Ref. [11] show hints that such correlations might exist.

6. Conclusions

The Chiral Magnetic Effect is a signature for $\mathcal{P}$- and $\mathcal{CP}$-violation on an event-by-event basis. It will be direct evidence for the existence of topologically nontrivial configurations of gauge fields, and therefore complement searches for instantons in scattering experiments [22].

Observation of the Chiral Magnetic Effect will be evidence for the existence of a quark gluon plasma, a phase in which matter is deconfined and chiral symmetry is restored. The reason is that if the quarks are confined, we cannot separate them. Moreover, if chiral symmetry is broken, quarks become effectively massive, which removes the necessary correlation between the spin and momentum of the quarks.

In order to obtain our results, we had to make some approximations. It would be desirable to improve our results to more obtain accurate predictions for the absolute magnitude and the dependence of the Chiral Magnetic Effect on the kind of nucleus, the beam energy and the impact parameter. Nevertheless, the Chiral Magnetic Effect is a very natural consequence of QCD in the presence of a strong magnetic field. It is therefore conceivable that it might be observed in heavy ion collisions.

Let us finally point out that the Chiral Magnetic Effect is in some sense similar to Electroweak Baryogenesis in the early universe. There a $\mathcal{C}$- and $\mathcal{CP}$-violating sphaleron transition induces via the axial anomaly a difference in the number of baryon plus lepton number [16]. In some scenarios this then could lead to the observed baryon asymmetry of the universe [23]. If however large magnetic fields were present in the early universe during the QCD phase transition, the Chiral Magnetic Effect itself might even have implications for cosmology [24].

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